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FIRST PRE-BOARD EXAMINATION (2023-24)

CLASS XII **MATHEMATICS** **TIME: 3 hrs**

Max Marks : 80

General Instructions:

1. This Question paper contains **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment (**4marks each**)** with sub-parts.

SECTION A
(Multiple Choice Questions)
Each question carries 1mark

1). Sum of the order and degree of the differential equation $\left(x + \frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)^2 + 1$ is

(a) 1 (b) 2 (c) 3 (d) 4

2). The corner points of the feasible region determined by the system of linear constraints are (0,3), (1,1) and (3,0). Let Z = px + qy, where p, q > 0. Condition on p and q so that the minimum of Z occurs at both the points (3,0) and (1,1) is

(a) 2p = q (b) p =2q (c) q = p (d) q = 3p

3) . If A is a square matrix such that $A^2 = A$, then the value of $7A - (I+A)^3$, where I is an identity matrix, is

(a) – I (b) I (c) A (d) A – I

4). The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$, is

(a) log y = kx (b) xy = k (c) y = kx (d) y = k log x

5). The region represented by the inequation system $x, y \geq 0$, $y \leq 6, x + y \leq 3$ is

(a) unbounded in the first quadrant (b) unbounded in first and second quadrants
(c) bounded in first quadrant containing the origin
(d) bounded in first quadrant not containing the origin

6). A problem in mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the event of their solving the problem are independent, then the probability that the problem will be solved, is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

7). If the area of the triangle with vertices (-3,0) ,(3,0) and (0,k) is 9 sq units, then the value(s) of k will be
(a) ± 9 (b) ± 3 (c) ± 8 (d) $\pm$ 4

8). Let A be a 3×3 matrix such that $|adj\ A|$ = 64. Then $|A|$ is equal to
(a) 8 only (b) -8 only (c) 64 (d) 8 or -8

9). If A = $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and 2A + B is a null matrix, then B is equal to

(a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & -4 \\ -5 & -2 \end{bmatrix}$

10). If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$ = P + Q , where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to

(a) $\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & \frac{5}{2} \\ \frac{-5}{2} & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & \frac{-5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$

11). If f(x) is continuous at x=1 and f(x) = $\begin{cases} 4x^2 + 3bx, & x \neq 1 \\ 5x - 4, & x = 1 \end{cases}$ then the value of ‘b’ is

(a) 3 (b) - 1 (c) 1 (d) - 3

12). If y = e^{-x}, then $\frac{d^2y}{dx^2}$ is equal to
(a) y (b) - y (c) x (d) – x.

13). If $\frac{d}{dx}[f(x)] = ax + b \ and \ f(0) = 0$, then f(x) is equal to
(a) a + b (b) b (c) $\frac{ax^2}{2} + bx$ (d) $\frac{ax^2}{2} + bx + c$

14). If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

15). In a triangle OAC, if B is the midpoint of side AC and $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}, then\ \overrightarrow{OC}\ is\ equal\ to$.
(a) $\vec{a} + \vec{b}$ (b) $\frac{\vec{a}+\vec{b}}{2}$ (c) $2\vec{a} + \vec{b}$ (d) $2\vec{b} - \vec{a}$

16). The value of $(\hat{\imath} \times \hat{\jmath}).\hat{k} + \hat{\imath}.\hat{\jmath}$
(a) - 1 (b) 0 (c) 2 (d) 1

17). The two lines $\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k})$+ μ $(2\hat{\imath} + 3\hat{\jmath} - 6\hat{k})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k})$+ α $(6\hat{\imath} + 9\hat{\jmath} - 18\hat{k})$ are
(a) parallel (b) intersecting (c) perpendicular (d) skew

18). The direction cosines of a vector equally inclined to the axes OX, OY and OZ are
(a) (1,1,1) (b) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (c) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (d) (1,2,3)

ASSERTION-REASON BASED QUESTIONS:

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19). Assertion (A) : Let A = {2,4,6} and B = {3,5,7,9} and defined a function f = {(2,3),(4,5),(6,7)} from A to B. Then, f is not onto.

Reason(R): A function f : A → B is onto, if every element of B is the image of some element of A under f.

20). Assertion (A) : If f(x) = log(sin x), x > 0 is strictly decreasing in $\left(\pi, \frac{3\pi}{2}\right)$

Reason(R): If $f'(x) > 0$, then f(x) is strictly increasing function.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2marks each

21). Find the principal value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$.

(OR)

Find the range of the function $f(x) = 2\sin^{-1} x + \frac{3\pi}{2}$

22). The total revenue in rupees received from the sale of 'x' units of a product is given by

$R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when x = 7.

23). Find the interval(s) in which the function f : R → R defined by $f(x) = x^2 e^{-x}$, is increasing

24). If $f(x) = 9x^2 + 12x + 2$, then find the minimum value of f(x).

(OR)

Find the maximum profit that a company can make , if the profit function is given by $P(x) = 72 + 42x - x^2$, where x is the number of units and P is the profit in rupees.

25). Evaluate :

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (x^3 + x\cos x + tan^5 x)\, dx$$

SECTION C

This section comprises of Short Answer type questions (SA) of 3marks each

26). If x = a (cos t + t sin t) and y = a (sin t – t cos t) , $0 < t < \frac{\pi}{2}$, find $\frac{d^2y}{dx^2}$.

27). Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$.

(OR)

Evaluate : $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx$.

28). Evaluate : $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}$ dx.

29). Solve the differential equation $\frac{dx}{dy} + x\cot y = 2y + y^2 \cot y$ where $y \neq 0$

(OR)

Solve the differential equation $\left[x\, sin^2\left(\frac{y}{x}\right) - y\right]$ dx + x dy =0

30) Solve the following linear programming problem graphically.

Minimize Z = 3 x + 9 y

Subject to

$x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$, $x, y \geq 0$

(OR)

Solve the following linear programming problem graphically.

Minimize Z = 50 x + 70 y

Subject to

$2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$

31). A random variable X has the following probability distribution, where k is some real number

X	0	1	2	otherwise
P(X)	k	2k	3k	0

(1) Determine the value of k

(2) Find P(X<2)

(3) Find P(X>2)

SECTION D

This section comprises of Long Answer-type questions (LA) of 5 marks each

32). Prove that the relation R on the set N×N defined by (a,b) R (c,d) ⇒ a + d = b + c, for every (a,b), (c,d) $\in N \times N$ is an equivalence relation.

(OR)

If R_1 *and* R_2 are equivalence relations in a set A , show that $R_1 \cap R_2$ is also an equivalence relation.

33). Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

34). Using matrix method, solve

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4\,,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1\,,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

35). Find the coordinates of the image of the point P(0,2,3) with respect to the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$

(OR)

Find the shortest distance between the lines whose equations are

$\vec{r_1}=\hat{\imath}+\hat{\jmath}+\lambda(2\hat{\imath}-\hat{\jmath}+\hat{k})$ and $\vec{r_2}=2\hat{\imath}+\hat{\jmath}-\hat{k}+\mu(3\hat{\imath}-5\hat{\jmath}+2\hat{k})$

SECTION E

This section comprises of 3 case-study/passage-based questions of 4 marks each. First two questions have three sub-parts (1),(2),(3) of marks 1,1,2 respectively. The third case study question has two sub-parts of 2 marks each.

36). Read the following passage and answer the questions given below.

A house is being constructed and a lot of planning is put into it. Now a person is confused about the window. He wants the window in the form of a rectangle surmounted by a semicircle such that the perimeter of the window is to be 10 metres. If radius of the semicircular portion is 'r' metres and height of the rectangular portion is 'x' metres, then

(1) Write a relation between x and r.
(2) Represent the area in terms of 'r'.
(3) Find the critical point, with respect to area, in terms of 'r'.

(OR)

(3) What are dimensions so that maximum light may enter the room?

(Note: Internal choice is for option 3)

37). A student of class XII wants to find the displacement of an object using the formula

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}\,t^2$ and $\vec{a} = \frac{\vec{F}}{m}$, $where\ \vec{u} = 2\hat{\imath}\ m/s$ and mass of the object is 2 kg. Force on the object are as (Newton unit)

$\overrightarrow{F_1} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, $\overrightarrow{F_2} = 2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

From the above information answer the following

(1) Find the net force on the object.

(2) Find the acceleration of the object.

(3) Find the unit vector perpendicular to both $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$.

(OR)

(3) Find the displacement in 2 seconds.

(Note: Internal choice is for option 3)

38). A company has two plants to manufacture TVs. The first plant manufactures 70% of the TVs and the rest are manufactured by the second plant. 80% of the TVs manufactured by the first plant are rated of standard quality, while that of the second plant only 60% are of standard quality. One TV is selected at random.

Based on the above information answer the following :

(i) Find the probability that the selected TV is of standard quality.

(ii) Find the probability that the TV is of standard quality, given that it was manufactured by the first plant.

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