Kendriya Vidyalaya Sangathan, Jaipur Region

First Pre-Board Exam 2023-24

CLASS- XII

SUBJECT- MATHEMATICS (041)

Time: 3 Hours **SET-A** **Max.marks: 80**

General Instructions:

1. This Question paper contains - five sections **A, B, C, D and E**. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4marks each) with sub parts.

SECTION A

(This section comprises of Multiple-choice questions (MCQ) of 1 mark each.)

1. For what value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & -3 \\ 1 & 2 \end{vmatrix}$ equal?

(a) 5	(b) 2	(c) ± 5	(d) ± 2

2. If A = $\begin{vmatrix} 2 & \lambda & -4 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$, then A^{-1} exists, if

(a) $\lambda = 2$	(b) $\lambda \neq 2$	(c) $\lambda = -2$	(d) $\lambda \neq -2$

3. If A is a skew- symmetric matrix then $B^T AB$ will be

(a) Unit matrix	(b) Symmetric matrix
(c) Skew symmetric matrix	(d) zero matrix

4. A function f: R -> R is defined by: $f(x) = \begin{cases} e^{-2x}, & x < \ln\frac{1}{2} \\ 4\ , & \ln\frac{1}{2} \leq x \leq 0 \\ e^{-2x}, & x > 0 \end{cases}$

Which of the following statements is true about the function at the point $x = \ln\frac{1}{2}$

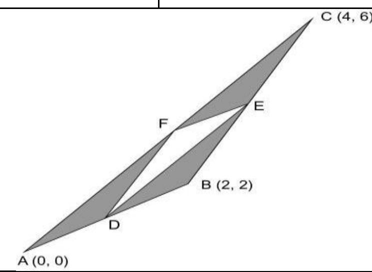
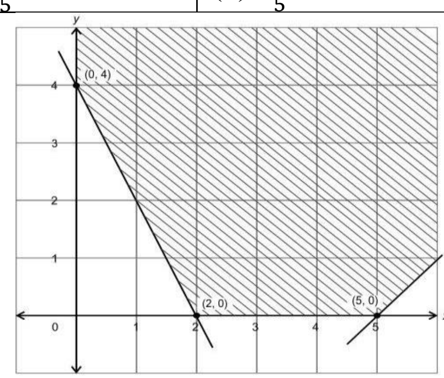
(a) $f(x)$ is not continuous but differentiable.
(b) $f(x)$ is continuous but not differentiable.
(c) $f(x)$ is neither continuous nor differentiable.
(d) $f(x)$ is both continuous as well as differentiable

5. In which of these interval is the function $f(x) = x^2 - 4x$ strictly decreasing?

(a)) $(-\infty, 0)$	(b)) $(0, 4)$	(c)) $(2, \infty)$	(d) $(-\infty, \infty)$

6. L1 and L2 are two skew lines. How many lines joining L1 and L2 can be drawn such that the line is perpendicular to both L1 and L2 ?

(a) exactly one	(b) exactly two	(c) zero	(d) infinitely many

7	If the value of a third order determinant is 12, then the value of the determinant formed by replacing each element by its cofactors will be			
	(a) 12	(b) 144	(c) -12	(d) 13
8	If $x^y = e^{x-y} then \frac{dy}{dx}$ is			
	(a) $\frac{1+x}{1+logx}$	(b) $\frac{1-\log x}{1+logx}$	(c) $\frac{\log x}{(1+\log x)^2}$	(d) not defined
9	The value of $\int_{-1}^{1} \log\left(\frac{2+x}{2-x}\right)$ is			
	(a) 0	(b) 1	(c) 2	(d) e
10	The points D, E and F are the mid-points of AB, BC and CA respectively. Where A (0,0) B (2,2) and C (4, 6) What is the area of the shaded region?			
	(a) 0.5 sq units	(b) 1.0 sq unit	(c) 1.5 sq unit	(d) 2.0 sq unit
11	Probability that A speaks truth is 4/5. A coin is tossed. A report that a head appears. The probability that actually there was head is			
	(a) $\frac{1}{2}$	(b) $\frac{1}{5}$	(c) $\frac{2}{5}$	(d) $\frac{4}{5}$
12	A linear programming problem (LPP) along with the graph of its constraints is shown here. The corresponding objective function is Minimize: Z = 3*x* + 2*y*. The minimum value of the objective function is obtained at the corner point (2, 0). The optimal solution of the above linear programming problem______________			
	(a) does not exist as the feasible region is unbounded. (b) does not exist as the inequality 3x + 2y < 6 does not have any point in common with the feasible region. (c) exists as the inequality 3x + 2y > 6 has infinitely many points in common with the feasible region. (d) exists as the inequality 3x + 2y < 6 does not have any point in common withthe feasible region.			

13 The feasible region of a linear programming problem is bounded. The corresponding objective function is Z = 6x - 7y.

The objective function attains ________________in the feasible region.

(a) only minimum
(b) only maximum
(c) both maximum and minimum
(d) either maximum or minimum but not both

14 Unit vector which makes equal angle with $\hat{\imath},\ \hat{\jmath}\ and\ \hat{k}$ is

(a) $\hat{\imath} + \hat{\jmath} + \hat{k}$	(b) $\hat{\imath}$
(c) $\hat{k}$	(d) none of these

15 The vector which is perpendicular to $\vec{a} = i - 2j + 3k$ and $\vec{b} = 2i + 3j - 5k$ is

(a) $\hat{\imath} - 11\hat{\jmath} + 7\hat{k}$	(b) $\hat{\imath} + 11\hat{\jmath} + 7\hat{k}$
(c) $\hat{\imath} - 11\hat{\jmath} - 7\hat{k}$	(d) $11\,\hat{\imath} + \hat{\jmath} + 7\hat{k}$

16 The value of ($\hat{k} \times \hat{\jmath}$). $\hat{\imath}$ + $\hat{\jmath}.\hat{k}$ is

(a) 0	(b) 1	(c) - 1	(d) 2 $\hat{\imath}$

17 The projection of $2\hat{\imath} - \hat{\jmath} - 4\,\hat{k}$ on vector $7\hat{k}$ is

(a) 4	(b) $\frac{28}{\sqrt{21}}$	(c) $-\frac{28}{\sqrt{21}}$	(d) 0

18 The difference of the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0\ is$$

(a) 1	(b) 2	(c) -1	(d) 0

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 & 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
(C) (A) is true, but (R) is false.
(D) (A) is false, but (R) is true.

19 **Assertion (A):** If n (A) = p and n (B) = q then the number of relations fromA to B is 2^{pq}

Reason (R) : A relation from A to B is a subset of A x B

20 **Assertion (A):** Degree of differential equation**:** $x - cos\ (dy/dx)$ = **0** is 1.

Reason (R): Differential equation $x - cos\ (dy/dx)$ = 0 can be converted in the polynomial equation of derivative.

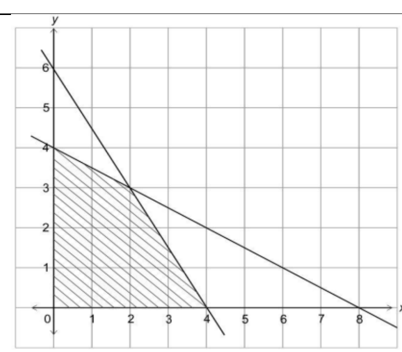
SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 markseach.)

21 Find the domain of the function $\cos^{-1}|x-1|$.

OR

Draw the graph of the following function: $y = 2\ \sin^{-1} x\ ,\ \ -\pi \le y \le \pi$

22	$If\ \ x = \sin\theta\ ,\quad y = \cos\theta\ \ find\ \dfrac{d^2y}{dx^2}\ \ when\ \theta = \dfrac{\pi}{4}$
23	Find the rate of change of the volume of a sphere with respect to its surface area when the radius is 2 cm.
24	Iqbal, a data analyst in a social media platform is tracking the number of active users on their site between 5 pm and 6 pm on a particular day. The user growth function is modelled by $N(t) = 1000\ e^{0.1\ t}$ where $N(t)$ represents the number of active users at time t minutes during that period. Find how fast the number of active users are increasing or decreasing at 10 minutes past 5 pm. OR
24	The population of rabbits in a forest is modelled by the function below: $\boldsymbol{P(t) = \dfrac{2000}{1+e^{-0.5\,t}}}$, where P represents the population of rabbits in t years. Determine whether the rabbit population is increasing or not, and justify your answer.
25	Evaluate : $\int (sin^{-1}x)^2\ dx$
	SECTION C (This section comprises of short answer type questions (SA) of 3 marks each)
26	If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$ then show that: $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$
27	Evaluate: $I = \int_0^{\frac{\pi}{2}} \ log\ sin\,x\,dx$
28	Evaluate $\int e^x\left(\dfrac{2\ +\ sin\ 2x}{1+\ cos2x}\right)dx$ OR Evaluate: $\int \dfrac{e^x}{\sqrt{5-4\,e^x-\ e^{2x}}}\,dx$
29	Find the particular solution of differential equation : $\dfrac{dy}{dx} + y\ cot\ x = 2x + x^2 \cot x$, given that $y = 0$ when $x = \dfrac{\pi}{2}$ OR Find the particular solution of the differential equation: $(x\,dy - y\,dx)\ y\ sin\ \dfrac{y}{x}\ = \ (\ y\ dx + x\ dy)\ x\ cos\dfrac{y}{x},$ given that $y = \pi$ when $x = 3$
30	In adjacent figure the feasible region of a maximization problem whose objective function is given by Z = 5x + 3y. i) List all the constraints the problem is subjected to. ii) Find the optimal solution of the problem. 

31	A company follows a model of bifurcating the tasks into the categories shown below At the beginning of a financial year, it was noticed that:

	URGENT	NOT URGENT
IMPORTANT	urgent and important	not urgent but important
NOT IMPORTANT	urgent but not important	not urgent and not important

- 40% of the total tasks were urgent and the rest were not
- half of the urgent tasks were important, and
- 30% of the tasks that were not urgent, were not important

What is the probability that a randomly selected task that is not important is urgent?

OR

31 Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group, without replacement. Find the probability distribution of selected persons who always speak the truth.

Section – D

(This section comprises of long answer type questions (LA) of 5 marks each)

32 The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segments. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones, but Sri Lanka and India are in the same time zone.

A relation R is defined on the set U = {All people on the Earth} such that
R = {(x, y) : the time difference between the time zones x and y reside in is 6 hours}.

i) Check whether the relation R is reflexive, symmetric and transitive.
ii) Is relation R an equivalence relation?

OR

$Let\ f : [1, \infty) \rightarrow [1, \infty)\ is\ given\ by\ f(x) = (x^2 + 1)^2 - 1$.

Check whether the function is bijective.

33 Find the product of $\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix}\begin{bmatrix} -4 & 18 & 12 \\ 0 & 4 & 2 \\ 2 & -6 & -4 \end{bmatrix}$

Hence solve the system of equations:

$$x - y + 2z = 1, \ 2y - 3z = 1, \ 3x - 2y + 4z = 2$$

34 Find the area of the region in the first quadrant enclosed by the $x - axis$, $the\ line\ y = x\ \ and\ \ the\ curve\ x^2 + y^2 = 32$.

35 Find the vector and Cartesian equations of the straight line passing through the point (-5, 7, -4) and in the direction of (3, -2, 1).
Also find the point where this straight line crosses the XY - plane.

OR

Given below are two lines $L_1\ and\ L_2$

$L_1 :\ 2x = 3y = -z$ and $L_2 :\ 6x = -y = -4z$

i. Find the angle between the two lines.
ii. Find the shortest distance between the two lines.

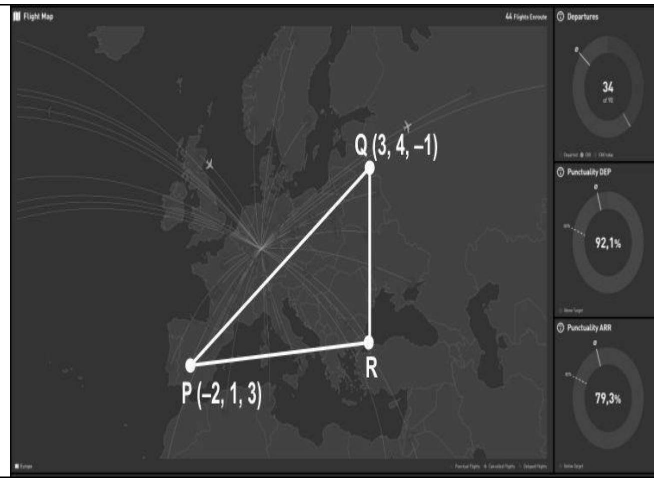
SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-questions. First two case study questions have three sub questions of marks 1, 1, 2 respectively. The third case study question has two sub questions of 2 marks each.)

36 The flight path of two airplanes in a flight simulator game are shown here. The coordinates of the airports P (-2, 1, 3) and Q (3, 4, -1) are given.

Airplane 1 flies directly from P to Q

Airplane 2 has a layover at R and then flies to Q.



The path of Airplane -2 from P to R can be represented by the vector $5\hat{\imath} + \hat{\jmath} - 2\hat{k}$

(Note: Assume that the flight path is straight and fuel is consumed uniformly throughout the flight.)

i) Find the vector that represents the flight path of Airplane 1.

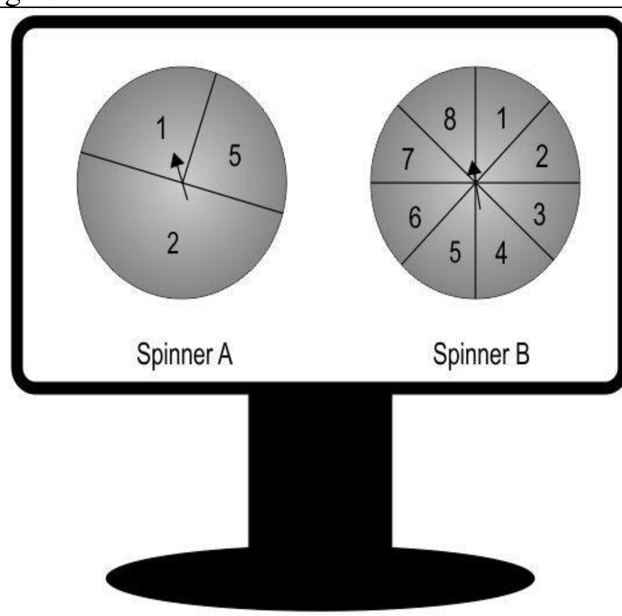
ii) Find the vector representing the path of Airplane 2 from R to Q.

iii) Find the angle between the flight paths of Airplane 1 and Airplane 2 just after take off?

OR

iii) Consider that Airplane- 1 started the flight with a full fuel tank.
Find the position vector of the point where one third of the fuel runs out if the entire fuel is required for the flight.

37 Rubiya, Taksh, Shanti, and Lilly entered a spinning zone for a fun game, but there is a twist: they don't know which spinner will appear on their screens until it is their turn to play. They may encounter one of the following spinners, or perhaps even both:



Different combinations of numbers will lead to exciting prizes. Below are some of the rewards they can win:

	➢ Get the number '5', from Spinner A and '8' from Spinner B, and you'll win a music player! ➢ You win a photo frame if Spinner A lands on a value greater than that of Spinner B!
	i) Taksh spuns both the spinners, A and B in one of his turns. What is the probability that Taksh wins a music player in his turn?
	ii) Lilly spuns spinner B in one of her turns. What is the probability that the number she got is even given that it is a multiple of 3?
	iii) Rubiya spuns both the spinners. What is the probability that she wins a photo frame? . OR
	iii) As Shanti steps up to the screen, the game administrator reveals that for her turn, the probability of seeing Spinner A on the screen is 65%, while that of Spinner B is 35%. What is the probability that Shanti gets the number '2'?
38	A cylindrical tank of fixed volume of 144 π m^3 is to be constructed with an open top to throw all the garbage in an orphanage. The manager of the orphanage called a contractor for the construction ensure that a tank to dispose off biodegradable waste can be constructed at a minimum cost. 
	i) Find the cost of the least expensive tank that can be constructed if it costs Rs. 80 per sq. m for base and Rs. 120 per sq. m for walls. ii) Find the radius and height as well.

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CLASS- XII

SUBJECT- MATHEMATICS (041)

SET - B

Time: 3 Hours **Max.marks: 80**

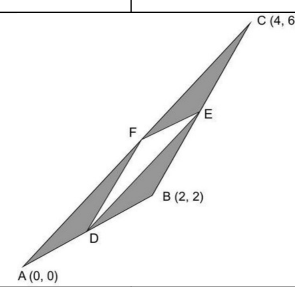
General Instructions:

1. This Question paper contains - five sections **A, B, C, D and E**. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(This section comprises of Multiple-choice questions (MCQ) of 1 mark each.)

1	Direction Ratios of a line perpendicular to XZ - plane are			
	(a) 1 , 0, 1	(b) 0 , 5 , 0	(c) 1 , 1 , 1	(d) $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$
2	A is a matrix of order 3 such that |adj A| = 7. Then find |A|			
	(a) 7	(b) 49	(c) $\sqrt{7}$	(d) $\frac{1}{7}$
3	$If\ y=\log\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right], then\ \frac{dy}{dx}\ is:$			
	(a) $sec\,x$	(b) $cosec\,x$	(c) tan x	(d) sec x tan x
4	$The\ value\ of \int_{-1}^{1}\log\left(\frac{2+x}{2-x}\right) is$			
	(a) 0	(b) 1	(c) 2	(d) e
5	For what value of x, are the determinants $\begin{vmatrix}2x & -3\\ 5 & x\end{vmatrix}$ and $\begin{vmatrix}10 & -3\\ 1 & 2\end{vmatrix}$ equal?			
	(a) 5	(b) 2	(c) ± 5	(d) ± 2
6	If A = $\begin{vmatrix}2 & \lambda & -4\\ 0 & 2 & 5\\ 1 & 1 & 3\end{vmatrix}$, then A^{-1} exists, if			
	(a) $\lambda=2$	(b) $\lambda\neq 2$	(c) $\lambda=-2$	(d) $\lambda\neq -2$

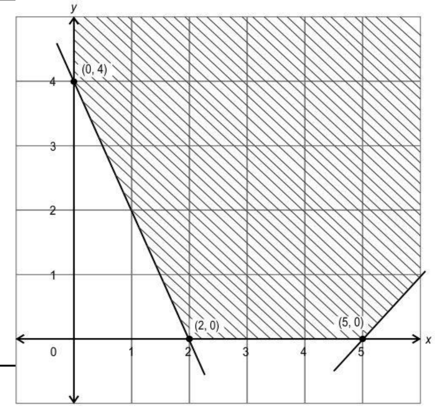
7	If A is a skew- symmetric matrix then $B^T AB$ will be			
	(a) Unit matrix		(b) Symmetric matrix	
	(c) Skew symmetric matrix		(d) zero matrix	
8	A function *f*: R -> R is defined by: $f(x)=\begin{cases} e^{-2x}, x<\ln\frac{1}{2} \\ 4, \ln\frac{1}{2}\leq x\leq 0 \\ e^{-2x}, x>0 \end{cases}$ Which of the following statements is true about the function at the point $x=\ln\frac{1}{2}$			
	(a) *f*(*x*) is not continuous but differentiable. (b) *f*(*x*) is continuous but not differentiable. (c) *f*(*x*) is neither continuous nor differentiable. **(d)** *f*(*x*) is both continuous as well as differentiable			
9	In which of these intervals is the function $f(x)=x^3+\frac{1}{x^3}, x>0$ is decreasing?			
	(a)) $[-1,1]$	(b)) $(-1,1)$	(c) $[-1,1]-\{0\}$	(d) $\{-1,1\}$
10	The points D, E and F are the mid-points of AB, BC and CA respectively. Where A (0,0) B (2,2) and C (4, 6) What is the area of the shaded region?			
	(a) 0.5 sq units	(b) 1.0 sq unit	(c) 1.5 sq unit	(d) 2.0 sq unit
11	Probability that A speaks truth is 4/5. A coin is tossed. A reports that a head appears. The probability that actually there was head is			
	(a) $\frac{1}{2}$	(b) $\frac{1}{5}$	(c) $\frac{2}{5}$	(d) $\frac{4}{5}$
12	The value of $(\hat{k}\times\hat{j}).\hat{i}+\hat{j}.\hat{k}$ is			
	(a) 0	(b) 1	(c) - 1	(d) $2\hat{i}$
13	The projection of $2\hat{i}-\hat{j}-4\hat{k}$ on vector $7\hat{k}$ is			
	(a) 4	(b) $\frac{28}{\sqrt{21}}$	(c) $\frac{-28}{\sqrt{21}}$	(d) 0
14	Integrating factor of the differential equation $x\frac{dy}{dx}-4x^2=2y$ is			
	(a) x^2	(b) $-x^2$	(c) $\frac{-1}{x^2}$	(d) $\frac{1}{x^2}$

15 A linear programming problem (LPP) along with the graph of its constraints is shown below. The corresponding objective function is

Minimize: $Z = 3x + 2y$.

The minimum value of the objective function is obtained at the corner point (2, 0).

The optimal solution of the above linear programming problem______________________________



(a) does not exist as the feasible region is unbounded.
(b) does not exist as the inequality $3x + 2y < 6$ does not have any point in common with the feasible region.
(c) exists as the inequality $3x + 2y > 6$ has infinitely many points in common with the feasible region.
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16 The feasible region of a linear programming problem is bounded. The corresponding objective function is $Z = 6x - 7y$.
The objective function attains __________________in the feasible region.

(a) only minimum
(b) only maximum
(c) both maximum and minimum
(d) either maximum or minimum but not both

17 If $\vec{a}$, $\vec{b}$ and ($\vec{a}$ + $\vec{b}$) are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is

(a) $\frac{2\pi}{3}$	(b) $\frac{\pi}{3}$	(c) $\frac{\pi}{6}$	(d) $\frac{5\pi}{6}$

18 The vector which is perpendicular to $\vec{a} = i - 2j + 3k$ and $\vec{b} = 2i + 3j - 5k$ is

(a) $\hat{i} - 11\hat{j} + 7\hat{k}$	(b) $\hat{i} + 11\hat{j} + 7\hat{k}$
(c) $\hat{i} - 11\hat{j} - 7\hat{k}$	(d) $11\hat{i} + \hat{j} + 7\hat{k}$

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 & 20, a statement of Assertion (A) is followed by a statement of Reason (R).
Choose the correct answer out of the following choices :
(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
(C) (A) is true, but (R) is false.
(D) (A) is false, but (R) is true.

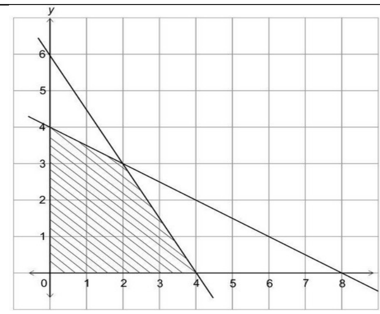
19 **Assertion (A):** If n (A) = p and n (B) = q then the number of relations from A to B is 2^{pq}

Reason (R) : A relation from A to B is a subset of A x B

20 **Assertion (A):** Degree of differential equation**:** $x - \cos(dy/dx) = \mathbf{0}$ is 1.

Reason (R): Differential equation $x - \cos(dy/dx) = \mathbf{0}$ can be converted in the polynomial equation of derivative.

	SECTION B (This section comprises of very short answer type-questions (VSA) of 2 marks each.)
21	Find the domain of the function $\cos^{-1}\lvert x-1\rvert$. OR Draw the graph of the following function: $y=2\sin^{-1}x, -\pi \le y \le \pi$
22	For what value of 'k' is the function $f(x)=\begin{cases} \frac{\sin 5x}{3x}+\cos x, & if \quad x\neq 0 \\ k, & if \quad x=0 \end{cases}$ continuous at x = 0?
23	Find the rate of change of the volume of a sphere with respect to its surface area when the radius is 2 cm.
24	Iqbal, a data analyst in a social media platform is tracking the number of active users on their site between 5 pm and 6 pm on a particular day. The user growth function is modelled by $N(t)=1000e^{0.1t}$ where N(t) represents the number of active users at time t minutes during that period. Find how fast the number of active users are increasing or decreasing at 10 minutes past 5 pm. OR The population of rabbits in a forest is modelled by the function below: $P(t)=\frac{2000}{1+e^{-0.5t}}$, where P represents the population of rabbits in t years. Determine whether the rabbit population is increasing or not, and justify your answer.
25	Evaluate : $\int_{-2}^{2}\lvert 1-x^2\rvert dx$
	SECTION C (This section comprises of short answer type questions (SA) of 3 marks each)
26	$If\ \sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$
27	Evaluate: $\int_{0}^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)\,dx$
28	$\int e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right)\Big\rvert dx$ OR Evaluate: $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$

29	Find the particular solution of differential equation : $\frac{dy}{dx}+y\cot x=2x+x^2\cot x$, given that $y=0$ when $x=\frac{\pi}{2}$ **OR** Find the particular solution of the differential equation: $(x\,dy-y\,dx)\,y\sin\frac{y}{x}=(y\,dx+x\,dy)\,x\cos\frac{y}{x}$, given that $y=\pi$ when $x=3$
30	In adjacent figure the feasible region of a maximization problem whose objective function is given by $Z = 5x + 3y$. i) List all the constraints the problem is subjected to. ii) Find the optimal solution of the problem. 
31	A company follows a model of bifurcating the tasks into the categories shown below At the beginning of a financial year, it was noticed that: (see table below)

	URGENT	NOT URGENT
IMPORTANT	urgent and important	not urgent but important
NOT IMPORTANT	urgent but not important	not urgent and not important

- 40% of the total tasks were urgent and the rest were not
- half of the urgent tasks were important, and
- 30% of the tasks that were not urgent, were not important

What is the probability that a randomly selected task that is not important is urgent?

OR

Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group, without replacement. Find the probability distribution of selected persons who always speak the truth.

Section – D

(This section comprises of long answer type questions (LA) of 5 marks each)

32 Find the product of $\begin{bmatrix}1 & 0 & 3\\ -1 & 2 & -2\\ 2 & -3 & 4\end{bmatrix}\begin{bmatrix}-4 & 18 & 12\\ 0 & 4 & 2\\ 2 & -6 & -4\end{bmatrix}$

Hence solve the system of equations:

$$x-y+2z=1, 2y-3z=1, 3x-2y+4z=2$$

33 Using integration, find the area of the region bounded by the triangle whose vertices are $(-1,2),(1,5)\wedge(3,4)$

34 The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segments. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones, but Sri Lanka and India are in the same time zone.

A relation R is defined on the set U = {All people on the Earth} such that

R = {(x, y)| the time difference between the time zones x and y reside in is 6 hours}.

i) Check whether the relation R is reflexive, symmetric and transitive.

ii) Is relation R an equivalence relation?

OR

$Let\, f:[1,\infty)\rightarrow[1,\infty)\, is\, given\, by\, f(x)=(x^2+1)^2-1.$

Check whether the function is bijective.

35 Find the vector and cartesian equations of the straight line passing through the point (-5, 7, -4) and in the direction of (3, -2, 1).

Also find the point where this straight line crosses the XY-plane.

OR

Given below are two lines $L_1 \wedge L_2$

$L_1: 2x=3y=-z$ and $L_2: 6x=-y=-4z$

i. Find the angle between the two lines.

ii. Find the shortest distance between the two lines..

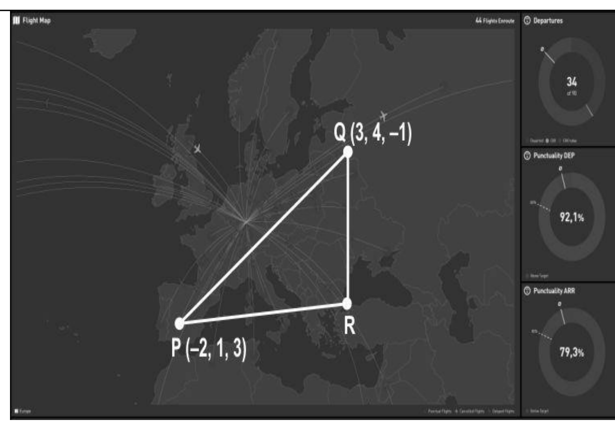
SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-questions. First two case study questions have three sub questions of marks 1, 1, 2 respectively. The third case study question has two sub questions of 2 marks each.)

36 The flight path of two airplanes in a flight simulator game are shown here. The coordinates of the airports P (-2, 1, 3) and Q (3, 4, -1) are given.

Airplane 1 flies directly from P to Q

Airplane 2 has a layover at R and then flies to Q.



The path of Airplane-2 from P to R can be represented by the vector $5\hat{i}+\hat{j}-2\hat{k}$

(Note: Assume that the flight path is straight and fuel is consumed uniformly throughout the flight.)

i) Find the vector that represents the flight path of Airplane 1.

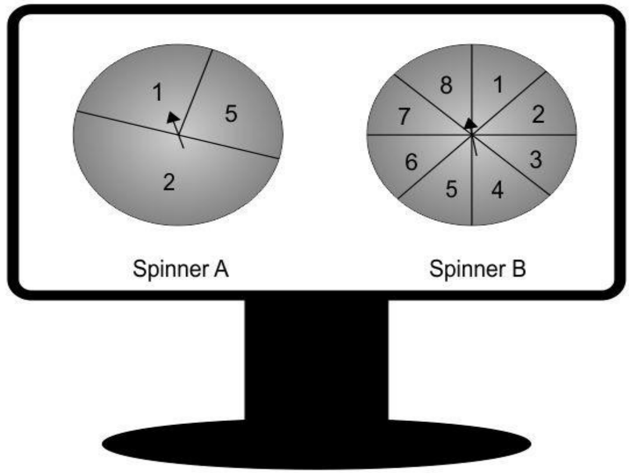
ii) Find the vector representing the path of Airplane 2 from R to Q.

iii) Find the angle between the flight paths of Airplane 1 and Airplane 2 just after take off?

OR

iii) Consider that Airplane- 1 started the flight with a full fuel tank. Find the position vector of the point where one third of the fuel runs out if the entire fuel is required for the flight.

37 Priya , Sahil, Annu, and Prachi entered a spinning zone for a fun game, but there is a twist: they don't know which spinner will appear on their screens until it is their turn to play. They may encounter one of the following spinners, or perhaps even both: Different combinations of numbers will lead to exciting prizes. Below are some of the rewards they can win:



- Get the number '5', from Spinner A and '8' from Spinner B, and you'll win a music player!
- You win a photo frame if Spinner A lands on a value greater than that of Spinner B!

i) Sahil spun both the spinners, A and B in one of his turns.

Find the probability that Sahil wins a music player in his turn?

ii) Prachi spun spinner B in one of her turns.

Find the probability that the number she got is even given that it is a multiple of 3? .

iii) Priya spun both the spinners.

What is the probability that she wins a photo frame?

OR

iii) As Annu steps up to the screen, the game administrator reveals that for her turn, the probability of seeing Spinner A on the screen is 65%, while that of Spinner B is 35%. Find the probability that Annu gets the number '2'?.

38 A cylindrical tank of fixed volume of 144 πm^3 is to be constructed with an open top to throw all the garbage in an orphanage. The manager of the orphanage called a contractor for the construction ensure that a tank to dispose off biodegradable waste can be constructed at a minimum cost.

	i) Find the cost of the least expensive tank that can be constructed if it costs Rs. 80 per sq. m for base and Rs. 120 per sq. m for walls. ii) Find the radius and height as well.

Kendriya Vidyalaya Sangathan, Jaipur Region

First Pre-Board Exam 2023-24

CLASS- XII

SUBJECT- MATHEMATICS (041)

SET - C

Time: 3 Hours **Max.marks: 80**

General Instructions:

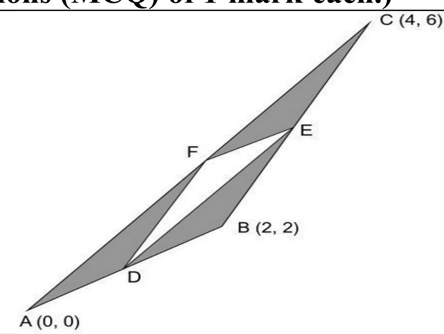
1. This Question paper contains - five sections **A, B, C, D and E**. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(This section comprises of Multiple-choice questions (MCQ) of 1 mark each.)

1 The points D, E and F are the mid-points of AB, BC and CA respectively.

Where A (0,0) B (2,2) and C (4, 6)
What is the area of the shaded region?



(a) 0.5 sq units	(b) 1.0 sq unit	(c) 1.5 sq unit	(d) 2.0 sq unit

2 Probability that A speaks truth is 4/5. A coin is tossed. A reports that a tail appears. The probability that actually there was head is

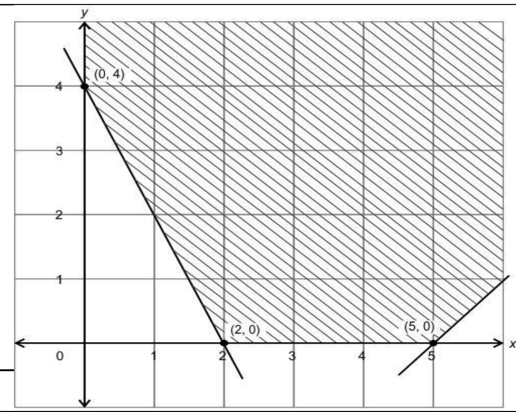
(a) $\frac{1}{2}$	(b) $\frac{1}{5}$	(c) $\frac{2}{5}$	(d) $\frac{3}{5}$

3 $If\ y=\log\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right], then \frac{dy}{dx} is:$

(a) $sec\,x$ (b) $cosec\,x$ (c) tan x (d) sec x tan x

4 $The\ value\ of \int_{-1}^{1}\log\left(\frac{2+x}{2-x}\right) is$

(a) 0	(b) 1	(c) 2	(d) e

5	A linear programming problem (LPP) along with the graph of its constraints is shown below. The corresponding objective function is Minimize: $Z = 3x + 2y$. The minimum value of the objective function is obtained at the corner point (2, 0). The optimal solution of the above linear programming problem____			
	(a) does not exist as the feasible region is unbounded. (b) does not exist as the inequality $3x + 2y < 6$ does not have any point in common with the feasible region. (c) exists as the inequality $3x + 2y > 6$ has infinitely many points in common with the feasible region. (d) exists as the inequality $3x + 2y < 6$ does not have any point in common with the feasible region.			
6	The feasible region of a linear programming problem is bounded. The corresponding objective function is $Z = 6x - 7y$. The objective function attains _________________in the feasible region. (a) only minimum (b) only maximum (c) both maximum and minimum **(d)** either maximum or minimum but not both			
7	$\vec{a} \wedge \vec{b}$ *are vectors such that* $\|\vec{a}\|=2, \|\vec{b}\|=3 \wedge \vec{a}.\vec{b}=4$*, then the value of* $\|\vec{a}-\vec{b}\|$ *is*:			
	(a) $\sqrt{5}$	(b) $\sqrt{21}$	(c) -1	(d) 1
8	The projection of $2\hat{i}-\hat{j}-4\hat{k}$ on vector $7\hat{k}$ is			
	(a) 4	(b) $\frac{28}{\sqrt{21}}$	(c) $\frac{-28}{\sqrt{21}}$	(d) 0
9	Integrating factor of the differential equation $x\frac{dy}{dx}-4x^2=2y$ is			
	(a) x^2	(b) $-x^2$	(c) $\frac{-1}{x^2}$	(d) $\frac{1}{x^2}$
10	Direction Ratios of a line perpendicular to xz plane are			
	(a) 1 , 0, 1	(b) 0 , 5 , 0	(c) 1 , 1 , 1	(d) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$
11	A is a matrix of order 3 such that $\|adj\ A\| = 7$. Then find $\|A\|$			

	(a) 7	(b) 49	(c) $\sqrt{7}$	(d) $\frac{1}{7}$
12	If A is a square matrix, such that $A^2=I$, then A^{-1} is equal to :			
	(a) A	(b) $2A$	(c) $A+I$	(d) I
13	If A = $\begin{vmatrix} 2 & \lambda & -4 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$, then A^{-1} exists, if			
	(a) $\lambda=2$	(b) $\lambda\neq 2$	(c) $\lambda=-2$	(d) $\lambda\neq -2$
14	Minimum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$ is			
	(a) 0	(b) -1	(c) $\frac{-1}{2}$	(d) $\frac{1}{2}$
15	A function *f*: R -> R is defined by: $f(x) = \begin{cases} e^{-2x}, & x < \ln\frac{1}{2} \\ 4 \;, & \ln\frac{1}{2} \le x \le 0 \\ e^{-2x}, & x > 0 \end{cases}$ Which of the following statements is true about the function at the point $x=\ln\frac{1}{2}$			
	(a) *f*(*x*) is not continuous but differentiable. (b) *f*(*x*) is continuous but not differentiable. (c) *f*(*x*) is neither continuous nor differentiable. **(d)** *f*(*x*) is both continuous as well as differentiable			
16	In which of these intervals is the function $f(x)=x^3+\frac{1}{x^3}, x>0$ is decreasing?			
	(a)) $[-1,1]$	(b)) $(-1,1)$	(c) $[-1,1]-\{0\}$	(d) $\{-1,1\}$
17	If $\vec{a}$, $\vec{b}$ and ($\vec{a}$ + $\vec{b}$) are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is			
	(a) $\frac{2\pi}{3}$	(b) $\frac{\pi}{3}$	(c) $\frac{\pi}{6}$	(d) $\frac{5\pi}{6}$
18	The vector which is perpendicular to $\vec{a}=i-2j+3k$ and $\vec{b}=2i+3j-5k$ is			
	(a) $\hat{i}-11\hat{j}+7\hat{k}$		(b) $\hat{i}+11\hat{j}+7\hat{k}$	
	(c) $\hat{i}-11\hat{j}-7\hat{k}$		(d) $11\hat{i}+\hat{j}+7\hat{k}$	
	ASSERTION-REASON BASED QUESTIONS In the following questions 19 & 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices : (A) Both (A) and (R) are true and (R) is the correct explanation of (A). (B) Both (A) and (R) are true, but (R) is not the correct explanation of (A). (C) (A) is true, but (R) is false. (D) (A) is false, but (R) is true.			

19	**Assertion (A):** If n (A) = p and n (B) = q then the number of relations from A to B is 2^{pq} **Reason (R)** : A relation from A to B is a subset of A x B
20	**Assertion (A):** Degree of differential equation**:** $x-\cos(dy/dx)=\mathbf{0}$ is 1. **Reason (R):** Differential equation $x-\cos(dy/dx)=\mathbf{0}$ can be converted in the polynomial equation of derivative.

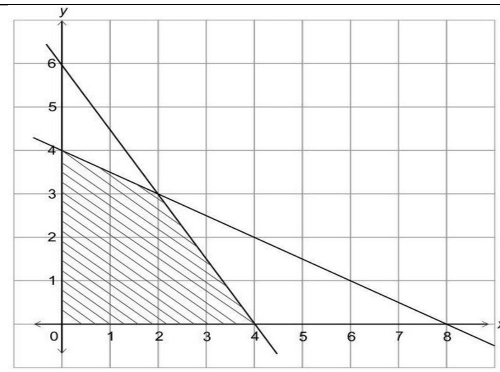
	<u>SECTION B</u> (This section comprises of very short answer type-questions (VSA) of 2 marks each.)
21	Evaluate : $\int_{-2}^{2} \lvert 1-x^2 \rvert dx$
22	Find the domain of the function $\cos^{-1}\lvert x-1 \rvert$. OR Draw the graph of the following function: $y=2\sin^{-1}x, -\pi \le y \le \pi$
23	If the circumference of circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius.
24	Find the value (s) of k so that the following function $f(x)=\begin{cases} \frac{1-\cos kx}{x\sin x}, if\ x\neq 0 \\ \frac{1}{2}, if\ x=0 \end{cases}$, is continuous at x = 0.
25	Iqbal, a data analyst in a social media platform is tracking the number of active users on their site between 5 pm and 6 pm on a particular day. The user growth function is modelled by $N(t)=1000\,e^{0.1t}$ where *N*(*t*) represents the number of active users at time *t* minutes during that period. Find how fast the number of active users are increasing or decreasing at 10 minutes past 5 pm. OR The population of rabbits in a forest is modelled by the function below: $P(t)=\frac{2000}{1+e^{-0.5t}}$ **,** where P represents the population of rabbits in t years. Determine whether the rabbit population is increasing or not, and justify your answer.
	<u>SECTION C</u> (This section comprises of short answer type questions (SA) of 3 marks each)

26 In adjacent figure the feasible region of a maximization problem whose objective function is given by

$Z = 5x + 3y$.

i) List all the constraints the problem is subjected to.

ii) Find the optimal solution of the problem.



27 Find the general solution of the differential equation

$$e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$$

OR

Find the general solution of the differential equation :- $x^2\frac{dy}{dx} = x^2 - 2y^2 + xy$

28 $If\ (x-a)^2 + (y-b)^2 = c^2,\ for\ c > 0,\ prove\ that$ $\dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is independent of a and b .

30 $\int e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right) dx$

OR

Evaluate: $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\, dx$

31 A company follows a model of bifurcating the tasks into the categories shown below

At the beginning of a financial year, it was noticed that:

	URGENT	NOT URGENT
IMPORTANT	urgent and important	not urgent but important
NOT IMPORTANT	urgent but not important	not urgent and not important

- 40% of the total tasks were urgent and the rest were not
- half of the urgent tasks were important, and
- 30% of the tasks that were not urgent, were not important

What is the probability that a randomly selected task that is not important is urgent?

What is the probability that a randomly selected task that is not important is urgent?

OR

Out of a group of 50 people, 20 always speak the truth. Two persons are selected at

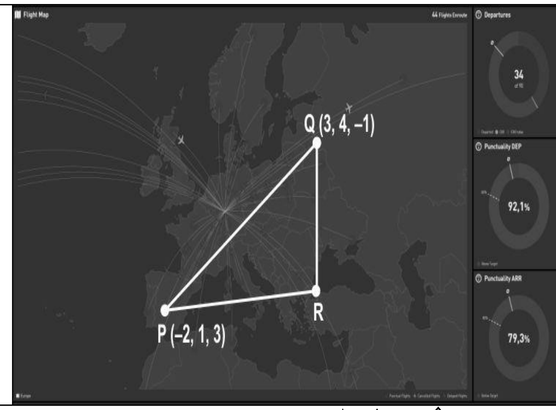
	random from the group, without replacement. Find the probability distribution of selected persons who always speak the truth.
	<u>Section – D</u> **(This section comprises of long answer type questions (LA) of 5 marks each)**
32	Find A^{-1}, If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Use the result to solve the following system of linear equation: $x + y + 2z = 0$: $x + 2y - z = 9$: $x - 3y + 3z = -14$
33	Using integration, find the area of the region bounded by the triangle whose vertices are $(-1,2),(1,5) \wedge (3,4)$
34	The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segments. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones, but Sri Lanka and India are in the same time zone. A relation R is defined on the set U = {All people on the Earth} such that R = {(x, y)| the time difference between the time zones x and y reside in is 6 hours}. i) Check whether the relation R is reflexive, symmetric and transitive. ii) Is relation R an equivalence relation? OR Let $f : R^{+} \rightarrow [-9,\infty)$ be a function defined as : $f(x) = 5x^2 + 6x - 9$ Show that f(x) is bijective.
35	Find the vector and Cartesian equations of the straight line passing through the point (-5, 7, -4) and in the direction of (3, -2, 1). Also find the point where this straight line crosses the XY-plane. OR Given below are two lines $L_1 \wedge L_2$ $L_1 : 2x = 3y = -z$ and $L_2 : 6x = -y = -4z$ i. Find the angle between the two lines. ii. Find the shortest distance between the two lines.
	SECTION E
	(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-questions. First two case study questions have three sub questions of marks 1, 1, 2 respectively. The third case study question has two sub questions of 2 marks each.)

36

The flight path of two airplanes in a flight simulator game are shown here. The coordinates of the airports P (-2, 1, 3) and Q (3, 4, -1) are given.

Airplane 1 flies directly from P to Q

Airplane 2 has a layover at R and then flies to Q.



The path of Airplane-2 from P to R can be represented by the vector $5\hat{i}+\hat{j}-2\hat{k}$

(Note: Assume that the flight path is straight and fuel is consumed uniformly throughout the flight.)

i) Find the vector that represents the flight path of Airplane 1.

ii) Find the vector representing the path of Airplane 2 from R to Q.

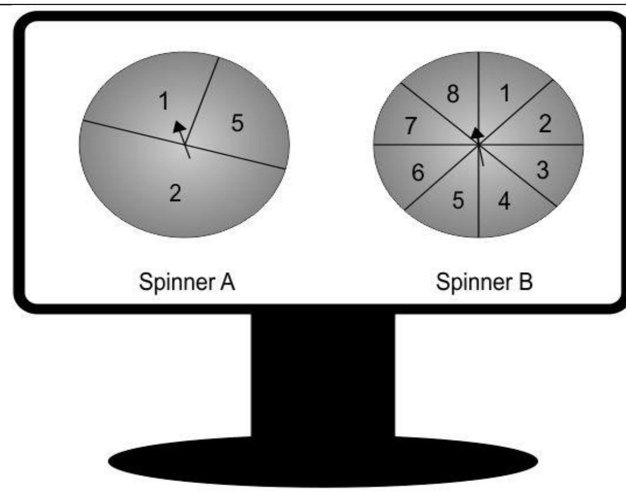
iii) Find the angle between the flight paths of Airplane 1 and Airplane 2 just after take off?

OR

iii) Consider that Airplane- 1 started the flight with a full fuel tank.
Find the position vector of the point where one third of the fuel runs out if the entire fuel is required for the flight.

37

Mamta, Rahul, Shreya, and Preeti entered a spinning zone for a fun game, but there is a twist: they don't know which spinner will appear on their screens until it is their turn to play. They may encounter one of the following spinners, or perhaps even both:
Different combinations of numbers will lead to exciting prizes. Below are some of the rewards they can win:



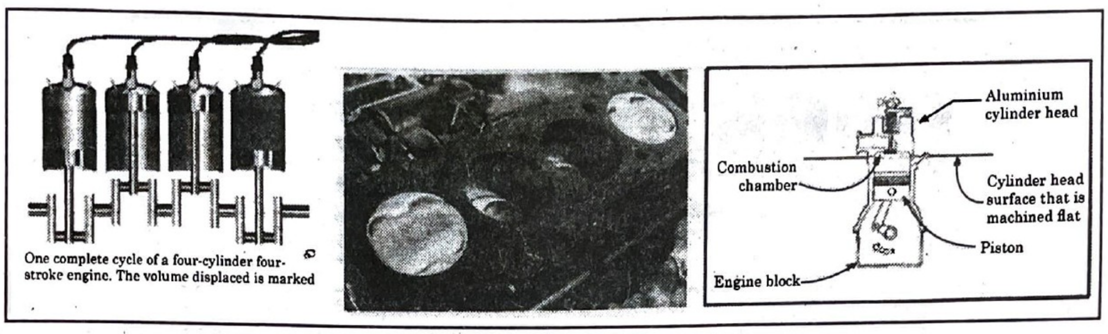
- Get the number '5', from Spinner A and '8' from Spinner B, and you'll win a music player!
- You win a photo frame if Spinner A lands on a value greater than that of Spinner B!

i) Rahul spun both the spinners, A and B in one of his turns.

Find the probability that Rahul wins a music player in his turn?

ii) Preeti spun spinner B in one of her turns.
Find the probability that the number she got is even given that it is a multiple of

<table>
<tr><td></td><td>3.</td></tr>
<tr><td></td><td>iii) Mamta spun both the spinners.
Find the probability that she wins a photo frame?
OR</td></tr>
<tr><td></td><td>iii) As Shreya steps up to the screen, the game administrator reveals that for her turn, the probability of seeing Spinner A on the screen is 65%, while that of Spinner B is 35%.
What is the probability that Shreya gets the number '2'?</td></tr>
<tr><td>38</td><td>Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore. The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi\ cm^2$



Based on the above information, answer the following questions :</td></tr>
<tr><td></td><td>(i) Find $\dfrac{dV}{dr}$
(ii) Find the radius of cylinder when its volume is maximum.</td></tr>
</table>