KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION
FIRST PRE-BOARD EXAMINATION 2023-24
CLASS XII
MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(This section comprises of Multiple-choice questions (MCQ) of 1 mark each.)

1. The function $\boldsymbol{f}: \boldsymbol{R} \rightarrow \boldsymbol{R}$ *is defined by* $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{x}^2$ is — 1

(a) One-One but not Onto (b) Onto but not One-one

(c) Both One-One and Onto (d) Neither One-One nor Onto

2. If $A = \begin{pmatrix} 1 & 2x & 5 \\ 8 & 3 & 12 \\ 5 & 4y & 7 \end{pmatrix}$ is a symmetric matrix then the value of $\boldsymbol{x}+\boldsymbol{y}$ is — 1

(a) 4 (b) 3 (c) 7 (d) 1

3. If $\begin{vmatrix} 2x & 3 \\ 4 & x \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ 4 & 18 \end{vmatrix}$ then find the value of x — 1

(a) 2 (b) 18 (c) 6 (d) ± 6

4. The determinant of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ is — 1

(a) 1 (b) 0 (c) 5 (d) 9

5. The derivative of the function $f(x) = x^x$ *with* respect to x is — 1

(a) x^x (b) $x^x . log x$

(c) $x^x(1 + log x)$ (d) $x^x(1 - log x)$

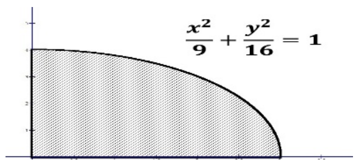
6. $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$ *is* — 1

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

7. The area of the shaded region in the given figure is $\left(\frac{x^2}{9} + \frac{y^2}{16} = 1\right)$ — 1

(a) 4π (b) 3π

(c) 12π (d) 6π



8. The function $f(x) = |x|$ *is* — 1

(a) Continuous at $x = 0$ but not differentiable at $x = 0$.

(b) Continuous at $x = 0$ and differentiable at $x = 0$.

(c) Neither continuous nor differentiable at x=0.

(d) Continuous at $x = 1$ and differentiable at $x = 1$.

9. A point $x = a$ is said to be critical point for the function $f(x)$ *if* 1

(a) $f(x)$ is differentiable at x=a. i.e. $f'(a) exist$.

(b) $f'(a) = 0$.

(c) $f'(a) = 0$ **or** $f(x)$ is not differentiable at $x = a$.

(d) $f''(a) = 0$.

10. If $f(x)$ is differentiable at c and $f(x)$ attains maximum value at c if identify the correct statement 1

(a) The sign of $f'(x)$ changes from negative to positive as x increasing through c.

(b) The sign of $f'(x)$ changes from positive to negative as x increasing through c.

(c) The sign of $f'(x)$ changes does not changes.

(d) The sign of $f''(x)$ changes from negative to positive as x increasing through c.

11. If $A = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is symmetric and Q is a skew symmetric matrix, then P is equal to 1

(a) $\begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & \frac{5}{2} \\ -\frac{5}{2} & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -\frac{5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$

12. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to: 1

(a) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

13. The sum of order and degree of the differential equation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^3=e^x$ is 1

(a) 2 (b) 3 (c) 5 (d) 4

14. Integrating factor of the deferential equation $\frac{dy}{dx} = x + y$ is 1

(a) e^y (b) e^{-y} (c) e^{-x} (d) e^x

15. The projection of the vector $\hat{i} - 2\hat{j} + \hat{k}$ on $3\hat{i} + 5\hat{j} + 7\hat{k}$ is 1

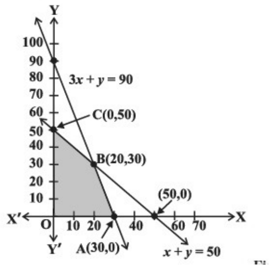
(a) 0 (b) $\sqrt{6}$ (c) $\sqrt{83}$ (d) $\sqrt{38}$

16. Two events A and B will be independent, then 1

(a) A *and* B' *are independent* (b) A' *and* B *are independent*

(c) A' *and* B' *are independent* (d) All of them correct

17. For the following LPP Maximize $z = 4x + y$ subject to the conditions $x + y \leq 50, 3x + y \leq 90, x \geq 0, y \geq 0$. The feasible solution for the given LPP as follows. Then the maximum value of z is 1



(a) 120 (b) 110

(c) 50 (d) 130

18. Let $\bar{a}$ and $\bar{b}$ are two vectors. In the following does not assure $\bar{a} \parallel \bar{b}$. 1

(a) $\bar{a} = \alpha\, \bar{b}$ (b) $\bar{a} = -\,\bar{b}$

(c) $\bar{a}$ and $\bar{b}$ are collinear (d) $|\bar{a}| = |\bar{b}|$

Instructions for Q19 & Q20 are as follows

Two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

(a) Both (A) and (R) are true and (R) is the correct explanation for (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation for (A).

(c) (A) is false but (R) is true.

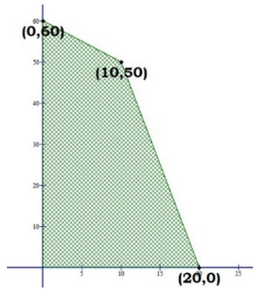
(d) Both (A) and (R) are false.

19. Let l, m, n are direction cosines of the line which makes angles with x, y and z axis respectively α, β and γ. 1

Assertion (A): $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Reason (R): $l^2 + m^2 + n^2 = 1$

20. Consider, the graph of constraints stated as linear inequalities as below: $5x + y \leq 100, x + y \leq 60, x, y \geq 0$. 1



Assertion (A): The points (10, 50), (0, 60), (10, 10) and (20, 0) are feasible solutions.

Reason (R): Points within and on the boundary of the feasible region represent feasible solutions of the constraints.

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$. 2

(or)

Find the value of $\tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

22. The radius r of a right circular cone is decreasing at the rate of 3 cm/minute and the height h is increasing at the rate of 2 cm/minute. When r = 9 cm and h = 6 cm, find the rate of change of its volume. 2

(or)

The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in rupees. Find the marginal revenue when x = 5, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.

23. Find the interval in which the function $f(x) = 3x^2 - 12x + 9$ is increasing. 2

24. Evaluate $\int \frac{e^x(x-3)}{(x-1)^3} dx$ 2

25. If $\bar{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\bar{b} = 2\hat{\imath} + \hat{\jmath}$ and $\bar{c} = 3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$, then find a vector perpendicular to both of the vectors $(\bar{a} - \bar{b})$ *and* $(\bar{b} - \bar{c})$. 2

(or)

If $\bar{a}, \bar{b}, \bar{c}$ are unit vectors such that $\bar{a} + \bar{b} + \bar{c} = \bar{0}$. Then find the value of $\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a}$

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Solve the following linear programming problem graphically : 3

Minimize : z = 3x + 9y When: $x + 3y \le 60, x + y \ge 10, x \le y, x \ge 0, y \ge 0$

27. $If\ x = a\,(\cos t + t \sin t) and\ y = a\,(\sin t - t\cos t), then\ find \frac{dy}{dx}.$ 3

(or)

If $y = (\tan^{-1} x)^2, then\ prove\ that\ (x^2+1)^2 y'' + 2x(x^2+1)y' - 2 = 0$

28. Evaluate $\int \frac{(x-1)}{(x-2)(x-3)}$dx 3

(or)

Evaluate $\int \sqrt{x^2 - 2x}\, dx$

29. Evaluate $\int_0^{\frac{\pi}{4}} \log(1 + tanx)\, dx$ 3

30. Solve the differential equation $x.logx.\frac{dy}{dx} + y = \frac{2}{x}.logx$. 3

(or)

Solve differential equation $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0.$

31. A coin is tossed three times. 3

Determine $P\left(\frac{E}{F}\right)$ *for the following.*

(i) E : head on third toss , F : heads on first two tosses

(ii) E : at least two heads , F : at most two heads

(iii) E : at most two tails , F : at least one tail

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Given below are two lines l_1 *and* l_2.

$l_1: \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ $\quad l_2: \bar{r} = (3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}) + \lambda(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

(i) Find the angle between the lines. 2

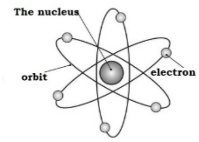
(ii) Find the shortest distance between the lines. 3

Show your work.

(or)

Find the co-ordinates of the foot of perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, –1, 3) and C (2, –3, –1). Show your work. 5

33. Electrons in an atom move around the nucleus in a path $\frac{x^2}{36} + \frac{y^2}{25} = 1$. Find the area covered by one of this path using integration. Show your work. 5



34. In a ground students are playing with a ball with their positions on a two dimensional plane. They pass the ball or related to each other from one player to another player if and only if the sum of the x-coordinate of the first student and the y-coordinate of the second student is equal to the sum of the y-coordinate of the first student and the x-coordinate of the second student. Verify whether the relation is 5

(i) Reflexive or not?

(ii) Symmetric or not?

(iii) Transitive or not?

(iv) Equivalence Relation or not?

Show your work.

(or)

Let $f: N \to N(\textit{Set of Natural Numbers})$ be defined by

$$f(n) = \begin{cases} n+1 & if \quad n \textit{ is an odd number} \\ n-1 & if \quad n \textit{ is an even number} \end{cases}.$$

(i) Check if f is one-one.

(ii) Check if f is onto.

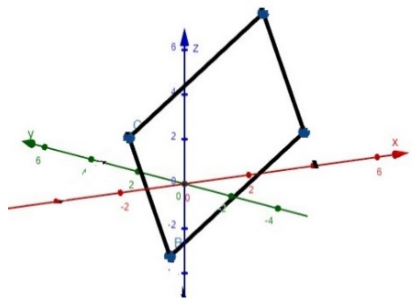
Show your work.

35. The theatre charges in a village ₹ 20 for children, ₹ 30 for students and ₹ 50 for adults. One day the movie “Life of Pi” is playing in that theatre. There are half as many as many adults as there are students and theatre was filled with full capacity of 400 members. If the total ticket sales was ₹ 13000, How many children, students, and adults are attended? 5

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-questions. First two case study questions have three sub questions of marks 1, 1, 2 respectively. The third case study question has two sub questions of 2 marks each.)

36. A student plots four points on a 3-D plane such that the four points form a parallelogram. He plots A (3,-1,2), B(1,2,-4) and C(-1,1,2).



(i) Find the coordinate of the fourth vertex. 1

(ii) Find the two adjacent sides of parallelogram in vector form. 1

(iii) Find area of the parallelogram using vector form of the two adjacent sides. 2

37. Of the students in a college, it is known that 70% reside in hostel and 30% reside outside the hostel. Previous year results report that 40% of hostellers attain A grade and 20% of those who reside outside attain A grade in the annual exam. At the end of the year, a student is chosen at random from the college.

(i) Express the given information in terms of probability. 1

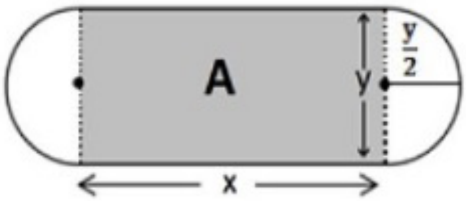
(ii) What is the probability that he has an A grade. 1

(iii) If the selected has an A grade, what is the probability that the student is a hosteler. 2

(OR)

(iii) If the selected student has an A grade, what is the probability that the student is residing outside? 2

38. An architect designs a building for a multi-national company. The floor consists of a rectangular region with semi-circular ends having a perimeter of 200m as shown below:



Based on the above information answer the following:

(i) If x and y represent the length and breadth of the rectangular region, then find the relation between the variables x and y. Also find the area of the rectangular region A expressed as a function of x? 2

(ii) Find the maximum value of area A? 2

***** **ALL THE BEST** *****

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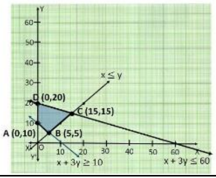
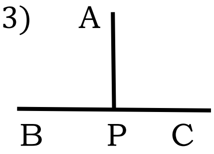
PRE-BOARD EXAMINATION 2023-24
CLASS XII
SUBJECT: MATHEMATICS (041)
Marking Scheme

Instruction:
All alternative methods are accepted and marks will be given according to the procedure.

1.d	2.c	3.d	4.b	5.c	6.d	7.b	8.a	9.c	10.b
11.b	12.c	13.b	14.c	15.a	16.d	17.a	18.d	19.a	20.a

21.	$\tan^{-1}\sqrt{3}=\frac{\pi}{3}$	---- ½ M
	$\cot^{-1}(-\sqrt{3})=\frac{5\pi}{6}$	---- ½ M
	$\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})=\frac{\pi}{3}-\frac{5\pi}{6}=-\frac{\pi}{2}$	---- 1M
	(or)	
	$\tan^{-1}(1)=\frac{\pi}{4}$	---- ½ M
	$\sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$	---- ½ M
	$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$	---- ½ M
	Answer : $\frac{\pi}{4}$	---- ½ M
22.	$v=\frac{1}{3}\pi r^2 h$ *and* $\frac{dv}{dt}=\frac{1}{3}\pi\left(r^2\frac{dh}{dt}+h.2r.\frac{dr}{dt}\right)$	---- ½ M
	Substitution of values	---- ½ M
	Simplification and Answer $-54\pi\ cm^2/\min$	---- 1M
	(or)	
	Marginal Revenue $\frac{dR}{dx}=6x+36$	---- ½ M
	Substitution of x value	---- ½ M
	Marginal revenue=6(5)+36=66	---- 1M
23.	$f'(x)=6x-12$	---- ½ M
	$x=2$	---- ½ M
	Given function is increasing in the interval $(2,\infty)$	---- 1M
24.	$\int\frac{e^x(x-3)}{(x-1)^3}dx=\int e^x\left[\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right]dx$	---- ½ M
	Formula	---- ½ M
	Required answer $\frac{e^x}{(x-1)^2}+c$	---- 1 M
25.	$(\bar{a}-\bar{b})=-\hat{\imath}+\hat{\jmath}+\hat{k}$	---- ½ M
	$(\bar{b}-\bar{c})=-\hat{\imath}+5\hat{\jmath}+5\hat{k}$	---- ½ M
	Required vector $4\hat{\jmath}-4\hat{k}$	---- 1 M
	(or)	
	$\bar{a},\bar{b},\bar{c}$ are unit vectors $\Rightarrow|\bar{a}|=|\bar{b}|=|\bar{c}|=1$	---- ½ M
	$(\bar{a}+\bar{b}+\bar{c}).(\bar{a}+\bar{b}+\bar{c})=0$	---- ½ M
	$\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}=-\frac{3}{2}$	---- 1 M

26.	For finding feasible region graphically	
		
	Graph	---- 1M
	Corner Points	---- 1M
	Minimum value at (5,5) is 60	---- 1M
27.	$\frac{dx}{dt} = atcost$	---- 1M
	$\frac{dy}{dt} = atsint$	---- 1M
	$\frac{dy}{dx} = tant$	---- 1M
	(or)	
	$y = (\tan^{-1} x)^2$	
	$\frac{dy}{dx} = 2\tan^{-1} x.\frac{1}{1+x^2}$	----1M
	$(x^2+1)y' = 2\tan^{-1} x$	
	$(x^2+1)y'' + 2xy' = \frac{2}{1+x^2}$	----1M
	$(x^2+1)^2y'' + 2x(x^2+1)y' - 2 = 0$	----1M
28.	$\frac{(x-1)}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$ & finding $A = -1$ & $B = 2$	---- 1 ½ M
	Required solution -ln(x - 2) + 2ln(x - 3) + c	---- 1 ½ M
	(or)	
	$\int \sqrt{x^2-2x}dx = \int \sqrt{(x-1)^2-1}dx$	---- 1M
	Formula	---- 1M
	Required answer $\frac{x-1}{2}\sqrt{x^2-2x} - \frac{1}{2}\ln\left\| (x-1) + \sqrt{x^2-2x}\right\| + c$	---- 1M
29.	$I = \int_0^{\frac{\pi}{4}} \log(1+tanx)\, dx$	
	Applying formula	---- ½ M
	$=\int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+tanx}\right) dx$	---- ½ M
	$=\int_0^{\frac{\pi}{4}} \log 2\, dx - \int_0^{\frac{\pi}{4}} \log(1+tanx)\, dx$	---- 1M
	$=\frac{\pi}{4} log2 - I$	
	$I=\frac{\pi}{8} log2$	---- 1M
30.	$\frac{dy}{dx} + \frac{1}{x.logx}.y = \frac{2}{x^2}$	---- ½ M
	Identification of linear DE	---- ½ M
	$IF = logx$	---- 1M
	General solution $y.logx = -\frac{2}{x}(1+logx) + c$	---- 1M
	(or)	
	$2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$	
	Writing $\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}}$	---- ½ M
	Identification of Homogeneous DE	---- ½ M
	$2e^v dv = -\frac{1}{y}dy$	---- 1 M
	$2e^{\frac{x}{y}} = -logy + c$	---- 1 M

31.	(i) $P\left(\frac{E}{F}\right) = \frac{1}{2}$ ---- 1 M (ii) $P\left(\frac{E}{F}\right) = \frac{3}{7}$ ---- 1 M (iii) $P\left(\frac{E}{F}\right) = \frac{6}{7}$ ---- 1 M
32.	Writing *points and drs of both the lines* ---- ½ M Writing formula ---- ½ M $(\overline{a_2} - \overline{a_1}) = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$ ---- ½ M $(\overline{b_1} \times \overline{b_2}) = -4\hat{\imath} - 6\hat{\jmath} - 8\hat{k}$ ---- ½ M Substitution and simplification distance $=\sqrt{116}\ units = 2\sqrt{29}\ units$ ---- 1 M Angle between two vectors formula ---- ½ M Angle between the lines $\theta = \cos^{-1}\left(\frac{10}{\sqrt{129}}\right)$ ---- 1 ½ M **(or)** Finding line equation $\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4}$ ---- 1 M Find general point P on the line $(2\lambda, -2\lambda - 1, -4\lambda + 3)$ ---- ½ M  Direction ratios of AP $(\mathbf{2\lambda + 1, -2\lambda - 9, -4\lambda - 1})$ ---- ½ M AP⊥BC $\Rightarrow \mathbf{2(2\lambda + 1) - 2(2\lambda - 9) - 4(-4\lambda - 1) = 0}$ ---- 1 M Finding λ=−1 ---- 1 M Coordinates of foot of perpendicular is (−2,1,7) ---- 1 M
33.	Figure ---- 1 M $y = \frac{5}{6}\sqrt{36 - x^2}$ ---- 1 M Area = $4 \int_0^6 \frac{5}{6}\sqrt{36 - x^2}\, dx$ ---- 1 M Area= $\frac{10}{3}\left(\frac{x}{2}\sqrt{36 - x^2} + \frac{36}{2}\sin^{-1}\left(\frac{x}{6}\right)\right)_0^6$ ---- 1 M Substitution and simplification 30π *square units* ---- 1 M
34.	Expressing given statement in mathematical form $(a, b)\ R\ (c, d)\ iff\ a + d = b + c$ ---- 1 M For reflexive ---- 1 M For symmetric ---- 1 M For transitive ---- 1 M For conclusion of equivalence relation ---- 1 M **(or)** For proving one-one (in all three cases) ---- 2 ½ M For proving onto ---- 2 ½ M
35.	Writing Equations ---- 1M Writing Matrix form ---- 1M Finding A^{-1} ---- 2M Finding number of children=100, Students=200, Adults=100 ---- 1M
36.	The coordinates of the fourth vertex are (1,−2,8) ---- 1 M The side $\overline{AB} = -2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ & $\overline{BC} = -2\hat{\imath} - \hat{\jmath} + 6\hat{k}$ ---- 1 M Area of the parallelogram= 28 Square units ---- 2 M
37.	$P(H) = \frac{7}{10}, P(O) = \frac{3}{10}, P\left(\frac{A}{H}\right) = \frac{4}{10}, P\left(\frac{A}{O}\right) = \frac{2}{10}$ ---- 1 M $P(A) = \frac{34}{100}$ ---- 1 M

	$P\left(\frac{H}{A}\right) = \frac{28}{34} = \frac{14}{17}$	---- 2 M
	(or)	
	(i) $\frac{1}{3}$	---- 1 M
	(ii) $\frac{1}{6}$	---- 1 M
	(iii) $\frac{2}{3}$	---- 1 M
38.	(i) $2x + \pi y = 200$	---- 1 M
	(ii) $\frac{2}{\pi}(100x - x^2)$	---- 1 M
	(iii) $\frac{5000}{\pi}\ m^2$	---- 2 M