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PRE BOARD EXAMINATION- 2023 – 24

Class- XII **Subject- Mathematics**

M.M. – 80 **Time- 3hours**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section –A

(Multiple Choice Questions)

Each question carries 1 mark

1. The value of x for which matrix A = $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ is a skew symmetric matrix:

   a. -2 c. 3
   b. 2 d. -3

2. If $\vec{a} = 7\hat{\imath} + \hat{\jmath} - 4\hat{k}$ and $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$, then projection of $\vec{a}$ on $\vec{b}$ is:

   a. 8/7 c. 3/4
   b. 7/8 d. 4/3

3. If $x + \frac{1}{x}$ = -2 , the principal value of $\sin^{-1} x$ is

   a. $\frac{\pi}{2}$ c. π
   b. $\frac{3\pi}{2}$ d. $-\frac{\pi}{2}$

4. Integrating factor of $\frac{dy}{dx}$ + 2y tan x = sin x is:

   a. Tan x c. $\text{Sec}^2 x$
   b. Sec x d. $\text{Tan}^2 x$

5. The function f: **N** → **N** defined by f(x) = 3x , x ∈ **N** is:

   a. One-one function c. Bijective function
   b. Ono function d. None of these

6. Point on the curve $y^2 = 8x + 8$ for which the abscissa and ordinate change at the same rate:

a. (1,2)
b. (2,1)
c. (4,1)
d. (1,4)

7. If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|A(adjA)|$is:

a. a^{27}
b. a^9
c. a^6
d. a^2

8. Function given by f(x) = $|x| - |x-1| + |x+2|$: is not differentiable at

a. 1 point
b. 2 points
c. 3 points
d. 4 points

9. The volume of a cube is increasing at the rate of 8 cm^3/sec. The rate of increase of its surface area when the length of an edge is 12 cm:

a. 2/3 Cm^2/sec.
b. 4/3 Cm^2/sec.
c. 5/3 Cm^2/sec.
d. 8/3 Cm^2/sec.

10. Vector form of the line $\frac{x-3}{-2} = \frac{y+2}{3} = \frac{z-1}{2}$ is:

a. $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k} + \lambda(3\hat{\imath} + 2\hat{\jmath} + \hat{k})$
b. $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k} + \lambda(-2\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$
c. $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k} + \lambda(-2\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$
d. $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$

11. $\int \text{Cos}(2x+3)$dx is:

a. $\frac{1}{2}$Sin (2x + 3) +C
b. -$\frac{1}{2}$ Sin (2x + 3) +C
c. Sin (2x + 3) +C
d. - Sin (2x + 3) +C

12. Let R be a relation defined on **Z** as follows : (x, y) ∈ R ⇔ $|x-y| \leq 1$, then R is:

a. Reflexive and transitive.
b. Reflexive and symmetric.
c. Symmetric and transitive.
d. Equivalence relation.

13. $\int_{-\pi/2}^{\pi/2} sin^7 x\, dx$ is equal to:

a. 0
b. 1
c. 1/6
d. 1/2

14. Order and degree of differential equation $2x^2\left(\frac{d^3y}{dx^3}\right)^2 - 3\left(\frac{dy}{dx}\right)^4 + y = 0$are:

a. 2,3
b. 3,2
c. 1,4
d. 1,2

15. If x = at^2 and y = 2at , then $\frac{dy}{dx}$ is :

a. 1/t
b. $-1/t^2$
c. -2/t
d. t

16. If P (A) = 0.8, P (B) = 0.5 and P (B|A) = 0.4, what is the value of P (A ∩ B)?

a. 0.32
b. 0.25
c. 0.1
d. 0.5

17. Value of $\tan^{-1}(1) + \cos^{-1}(\frac{1}{2}) + \sin^{-1}(\frac{1}{2})$ is:

a. $\frac{\pi}{4}$
b. $\frac{\pi}{2}$
c. $\frac{3\pi}{4}$
d. $\frac{3\pi}{2}$

18. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

a. 2/7
b. 3/7
c. 1/7
d. 1/3

Assertion reason based questions

Read both the statement carefully and choose the correct alternative from the following:

a. Both the statement A and R are correct and R is the correct explanation of A.
b. Both the statement A and R are correct and R is not the correct explanation of A.
c. The statement A is correct but R is false.
d. The statement A is false but R is correct.

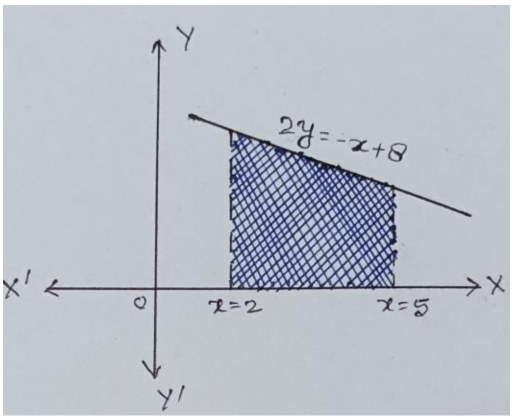
19. **Assertion(A):** Consider the two events E and F which are associated with the sample space of a random experiment. $P(E/F) = \frac{n(E \cap F)}{n(F)}$

Reason(R): $P(E/F) = \frac{P(E \cap F)}{P(F)}$

20. **Assertion(A)** : If the derivative function of x is $\frac{d}{dx}(x) = 1$, then its anti-derivatives or integral is $\int 1\,dx = x + c$

Reason(R): If $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$, then the corresponding integral of the function is $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$.

Section: B
(Very short questions)
Each question carries 2 marks



21. Using integration find area of shaded portion.

22. Find $\int \frac{1}{x+x\,logx}\,dx$

OR

Find $\int (4x+2)\sqrt{x^2+x+1}$ dx

23. Show that the function f(x) = $x^3 - 3x^2 + 4x$, x ∈ **R** is increasing on **R**.

24. A card is drawn at random from a pack of 52 cards and it is found to be a king card. Find the probability that the card drawn is a black card.

OR

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the selected oranges are good, then box is approved for sale otherwise it is rejected. Find the probability that a box containing 15 oranges, out of which 12 are good and 3 are bad ones, will be approved for sale.

25. Find derivative of x^x with respect to x.

Section: C

(Short questions)

Each question carries 3 marks

26. Show that the function f(x) defined by f(x) = $\begin{cases} \frac{4(1-\sqrt{1-x})}{x}, & if\ \ x<0 \\ 2, & if\ x=0 \\ \frac{\sin x}{x}+\cos x, & if\ \ x>0 \end{cases}$ is continuous at x = 0

OR

If $x^y = e^{x-y}$,Show that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$

27. Evaluate $\int \frac{1}{1+\cot x}$ dx.

OR

Evaluate $\int \frac{1}{1+\tan x}$ dx.

28. Find x, if $[x \quad -5 \quad -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

29. Using integration find area of the region bounded by the curve $y^2 = 4x$, the X- axis and the line x = 3.

OR

Using integration find area of the region bounded by the line y – 1 = x , the X- axis and the ordinates x = -2 and x = 3.

30. Find the value of λ and μ if $(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + \mu\hat{\jmath} + \lambda\,\hat{k}) = \vec{0}$

31. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with the probability 1/4. What is the probability that the student knows the answer given that he answered it correctly?

Section: D
(Long questions)
Each question carries 5 marks

32. If A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve

x + 2y + z = 4,

- x + y + z = 0,

x – 3y + z = 2.

33. Find the shortest distance between the lines: $\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$.

OR

Find the shortest distance between the lines $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $\frac{x-2}{2} = \frac{y+2}{1} = \frac{z+1}{2}$

34. Solve the following linear programming problem graphically:

Minimise and maximize Z = 5x + 10y subject to x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0.

Also show that maximum of Z occurs at two points.

35. Prove that the surface area of a solid cuboid, of square base and given volume, is minimum when it is a cube.

OR

Show that the rectangle of maximum area that can be inscribed in a circle is a square.

Section: E[4MARKS]

Case study / Source based/ integrated

36. An organization conducted bike race under 2 different categories-boys and girls. Totally there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let B = $\{b_1, b_2, b_3\}$ G=$\{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race.

Ravi decides to explore these sets for various types of relations and functions

Base on above information, answer the following questions.

A. Ravi wishes to form all the relations possible from B to G. How many such relations are possible? (1)

a. 2^6

b. 2^5

c. 2^3

d. 0

B. Let R: B→B be defined by R = {(x, y): x and y are students of same sex}, Then this relation R is_______ (1)

a. Equivalence relation

b. Reflexive relation only

c. Reflexive and symmetric but not transitive

d. Reflexive and transitive but not symmetric

C. Ravi wants to know among those relations, how many functions can be formed from B to G? (2)

a. 2^2

b. 2^{12}

c. 3^2

d. 2^3

OR

Let $R: B \to G$ be defined by R = { (b_1,g_1), (b_2,g_2),(b_3,g_1)}, then R is__________ (2)

a. One one function

b. Onto function

c. Bijective function

d. Neither one one nor onto function

37. A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let $\vec{a}, \vec{b}$ and $\vec{c}$ *be three* non zero vectors.

Base on above information, answer the following questions.

A. If $\vec{a}$, and $\vec{b}$ are such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then (2)

a. $\vec{a} \perp \vec{b}$

b. $\vec{a} \parallel \vec{b}$

c. $\vec{a} = \vec{b}$

d. None of these.

B If $\vec{a}$, and $\vec{b}$ are unit vectors and θ be angle between them, then $|\vec{a} - \vec{b}|$ is: (2)

a. $\text{Sin}\frac{\theta}{2}$

b. $2\text{Sin}\frac{\theta}{2}$

c. $2\text{Cos}\frac{\theta}{2}$

d. $\text{Cos}\frac{\theta}{2}$

38. Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = \mathbf{k(50 - y)}$ where x denotes the number of weeks and y the number of children who have been

given the drops.

Base on above information, answer the following questions.

A. Order of the above given differential equation. (1)

a. 0
b. 1
c. 2
d. 3

B Degree of the above given differential equation. (1)

a. 0
b. 1
c. 2
d. 3

C, Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = \mathbf{k(50 - y)}$? (2)

a. Variable separable method
b. Solving Homogeneous differential equation
c. Solving Linear differential equation
d. All of the above

OR

C The solution of the differential equation $\frac{dy}{dx} = \mathbf{k(50 - y)}$ is given by, (2)

a. log | 50 – y| = kx + C
b. - log | 50 – y| = kx + C
c. log | 50 – y| = log| kx |+ C
d. 50 – y = kx + C

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